

Mass transfer at smooth and rough surfaces in a circular Couette flow

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Abstract—An extended set of mass transfer measurements at smooth and rough surfaces in a circular Couette flow has been carried out in the range $20\,000 < Re < 900\,000$ and $770 < Sc < 8000$. Mass transfer measurements for smooth rotating cylinders are in agreement with previous results and confirm that the Sherwood number depends on the $1/3$ power of the Schmidt number. Mass transfer coefficients at rough cylinders show a similar behaviour with Reynolds number as that observed for duct flows and the experimental results for both systems can be generalized in the fully rough region for equally rough surfaces when the maximum velocity of the flow and the surface roughness height are taken as the characteristic velocity and length scale in the Stanton and Reynolds numbers.

INTRODUCTION

Mass transfer to a rotating cylinder under turbulent flow conditions is of interest in electrochemical studies about diffusion-controlled reactions. These studies are also of practical importance in electrochemical processes such as electrodeposition, coating, metal recovery, etc.

Mass transfer to or from rotating cylinders has been extensively studied by Eisenberg *et al.* [1, 2]. They proposed a modified Chilton and Colburn analogy of the form

$$Sh = 0.5 C_f Re Sc^{0.356} \quad (1)$$

to correlate their results in the range $112 \leq Re \leq 241\,000$ and $835 \leq Sc \leq 11\,490$. The friction values used in equation (1) were taken from Theodorsen and Regier [3]. Equation (1) has since been universally referred to in the literature [4] as the mass transfer correlation for rotating cylinders, and has even been claimed to be applicable to rough surface conditions [5]. Transfer processes at rough surfaces are not as well understood as they are for smooth walls and, for rotating rough cylinders, the problem is made worse by the scarcity of experimental information available.

Kappesser *et al.* [5] measured mass transfer rates at three cylinders roughened by means of staggered diamond knurls machined on their lateral surfaces. These authors found that equation (1) fitted their experimental electrochemical results when friction coefficients were evaluated from the coefficients determined by Theodorsen and Regier [3] in cylinders with sand glued to them, using the peak to valley distance

as the equivalent sand grain height. Nevertheless, other authors [6, 7] reported that this equivalence underestimates measured mass transfer rates when it is applied in conjunction with equation (1). It is well known [8] that for plane surfaces there is not a direct relation between the sand grain and the actual roughness height. Furthermore, the constancy of the Stanton number reported by some investigators [5–7] to occur for rotating rough cylinders beyond a given Reynolds number, contradicts the decrease of the Stanton with Reynolds number observed in pipe and duct flows over the fully rough region [9, 10].

The purpose of the present study is to determine mass transfer coefficients in smooth and in two rough rotating cylinders in the range $20\,000 < Re < 900\,000$ and $770 < Sc < 8000$, in order to assess the reliability of previous correlations and semi-empirical models for mass transfer at smooth cylinders, and to understand some characteristics of the transfer process over regular rough surfaces. To this end, mass transfer rates measured at the two rough surfaces considered are compared with the results obtained in a duct with plane surfaces of equal roughness.

EXPERIMENTAL

Electrochemical mass transfer measurements

Mass transfer rates were measured by the limiting-current technique, which has been widely employed in the past and is described in detail elsewhere [11, 12].

The redox system chosen in this study was formed by a 0.003 M solution of ferricyanide, a 0.015 M solution of ferrocyanide and by 1, 2 and 4 M solutions of sodium hydroxide as a supporting electrolyte to ensure negligible contribution of ionic migration to mass transfer. All solutions were prepared from dis-

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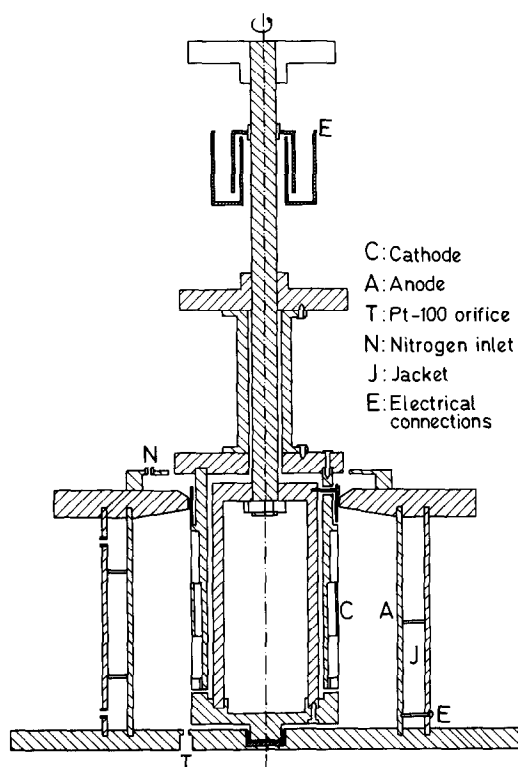


FIG. 1. Sketch of the apparatus.

located at an intermediate position of the inner rotating cylinder, between two Teflon rings, and the electrochemical solution entered by gravity into the gap between the cylinders.

A nitrogen atmosphere was maintained outside the electrochemical cell to prevent oxygen contamination during measurements. Water at the desired temperature was added to the jacket to control temperature, which was measured by means of a Pt-100 thermoresistance introduced periodically from the bottom plate up to the cathode level, at 2 mm from the inner wall. In order to ensure correct mass transfer measurements, a polarization curve was determined at the highest angular velocity to be studied for a given Schmidt number, obtaining always a good plateau for the limiting current.

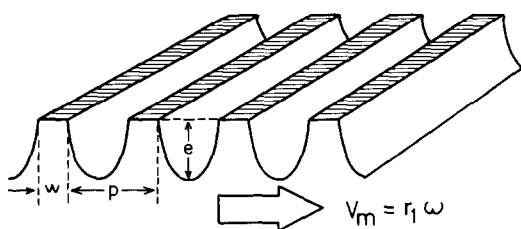


FIG. 2. Sketch of a V-shaped rough surface.

Table 1. Characteristic distances for V-shaped rough surfaces

Surface	p (mm)	e (mm)	w (mm)	p/e	w/e	r_1/e	d/e
R1	0.40	0.04	0.075	10.0	1.88	1750	1000
R2	0.76	0.23	0.19	3.3	0.83	304	174

RESULTS AND DISCUSSION

Mass transfer at smooth surfaces

The dimensionless mass transfer coefficients determined in the present study over the range of Schmidt numbers $770 < Sc < 8000$ are plotted as $Sh/Sc^{0.33}$ in Fig. 3 for $20\,000 < Re < 900\,000$. This figure also includes the data reported by Eisenberg *et al.* [1, 2] for $Re > 10\,000$. There is good agreement between both sets of data despite the fact that the Sherwood number has been normalized with respect to $Sc^{0.33}$ instead of with the Schmidt number dependence given by equation (1). The 0.356 exponent in this equation, recently corroborated by Pang and Ritchie [6], was not obtained in either case from direct multivariable correlation of experimental data and appears to be the result of a compromise taken by Eisenberg *et al.* [1, 2] in order to fit their data with a Chilton and Coulburn type of analogy—with friction factor values taken from Theodorsen and Regier [3].

A multivariable regression of the present experimental data yields the following correlations

$$Sh = 0.0987 Re^{0.70} Sc^{0.33}; \quad Re > 140\,000 \quad (4)$$

and

$$Sh = 0.0138 Re^{0.866} Sc^{0.33}; \quad Re > 140\,000. \quad (5)$$

In both equations a 0.33 ± 0.005 power dependence of the dimensionless mass transfer coefficients on the Schmidt number is obtained. This expected result, also published previously in the literature [15, 16], is in accordance with the dependence reported for the majority of experimental studies on turbulent mass transfer in Poiseuille or Couette type of flows at high Schmidt numbers. Furthermore, it should be noted that the scatter of the data of Eisenberg *et al.* [1, 2] plotted in Fig. 3 as $Sh/Sc^{0.33}$ does not improve when Sherwood numbers are normalized according to equation (1).

The original work of Eisenberg *et al.* [1, 2] and, therefore, the form of the modified Chilton and Coulburn equation (1) proposed by these authors, was strongly influenced by the values assigned to the friction coefficients, which at that time and for many years were only available from the work of Theodorsen and Regier [3]. Recently, Nakamura *et al.* [17] have determined friction factors from Reynolds shear stress and mean velocity gradient measurements. They have also estimated these coefficients from the mean velocity measurements reported by Smith and Townsend [18]. However, there is still large scatter between data

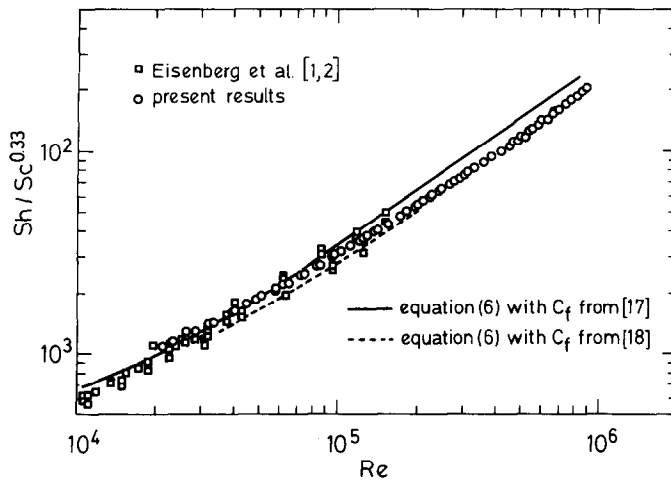


FIG. 3. Variation of the dimensionless mass transfer coefficients with Reynolds number at smooth rotating cylinders.

from different authors and no conclusive friction factor values may yet be given. As a consequence of all these facts, which limit the validity of equation (1), it seems necessary to look for a mass transfer equation that, on a phenomenological basis, could incorporate the $Sc^{1/3}$ dependence found experimentally.

Mass transfer processes at smooth surfaces in circular Couette flows are expected to be affected by curvature effects. Bradshaw [19] proposed an analogy between these effects and those of buoyancy. This analogy was used by Smith and Greif [16] to solve the mass conservation in a circular Couette flow, obtaining for high Schmidt numbers

$$Sh = 0.827(b/Sc_i)^{1/3}(C_f/2)^{1/2} \times Re Sc_i^{1/3}[1 + 40/(C_f Re)]^{2/3} \quad (6)$$

where b is the proportionality coefficient in the eddy diffusivity distribution for plane surfaces

$$(D_i/\nu)Sc_i = by_+^3. \quad (7)$$

In the present study the value $b/Sc_i = 6.5 \times 10^{-4}$, evaluated from the mass transfer measurements in pipe and duct flow [20, 14], has been adopted. All rotational effects on mass transfer are included in equation (6) by the factor $[1 + 40/(C_f Re)]^{2/3}$, which vanishes as the Reynolds number increases and can be omitted for $Re > 10^5$.

The mass transfer coefficients predicted by equation (6), using the friction coefficients measured by Nakamura *et al.* [17] and those estimated by these authors from Reynolds shear stress and velocity values reported by Smith and Townsend [18], are also shown in Fig. 3. Both predictions encompass the present data and the observed deviations may be considered reasonable if the scatter of both sets of friction coefficients is taken into account. Predictions obtained with the friction values reported by Theodorsen and Regier [3] and which are not included in Fig. 3, are in

agreement with the experimental mass transfer coefficients at low Reynolds numbers and about 30% above data at the highest Reynolds number considered. These results indicate the need for more reliable friction factor values and eddy diffusivity distributions in circular flows, before final conclusions can be drawn with respect to equation (6).

Mass transfer at rough surfaces

The mass transfer coefficients determined in surfaces R1 and R2 are presented in Fig. 4 as $Sh/Sc_i^{0.33}$ vs Re . The data has been normalized with respect to $Sc_i^{0.33}$ to illustrate deviations from the Schmidt number dependence obtained in smooth surfaces. All calculations have been based on the projected area because the transfer of matter between the bulk and the surface occurs through this area.

Figure 4 shows that for $Re < 1.3 \times 10^5$ transfer rates in the less rough surface R1 ($p/e = 10$) are equal to those obtained for smooth surface conditions, justifying the use of projected areas instead of the 20% greater contact area. Beyond this Reynolds number, transition from smooth to rough conditions begins and transfer rates are affected by surface roughness. Sherwood numbers increase faster with Reynolds than for the smooth case, this increase being higher with increasing Schmidt number. The trend of the data suggests that fully rough conditions have been barely reached for $Sc = 770$, due to the limitation in the maximum angular velocity of 1000 r.p.m. that could be attained without air entrainment in the present apparatus. Extrapolation of the data measured at $Sc = 2100$, yields a Schmidt number dependence of $Sh \propto Sc_i^{0.39}$. This result is consistent with the $Sh \propto Sc_i^{0.42}$ reported by Dawson and Trass [10] for V-shaped grooves with $p/e \approx 3.75$ in a duct flow, if it is considered that the increase in p/e from 3.75 to 10.0,

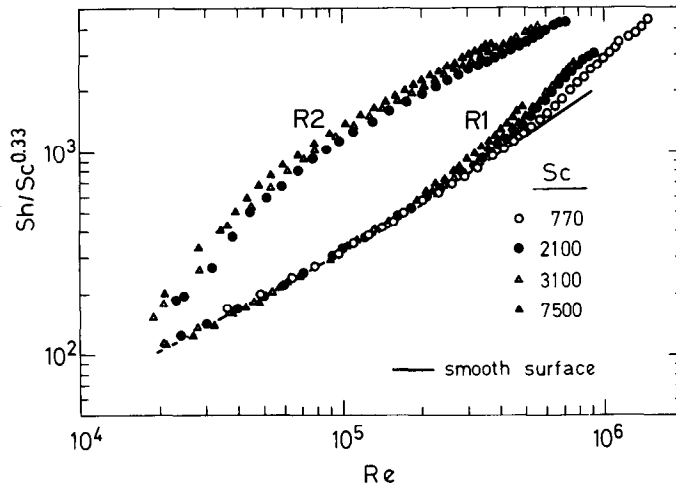


FIG. 4. Variation of the dimensionless mass transfer coefficients with Reynolds number at rough surfaces R1 and R2.

caused mainly by a drop in roughness height for the R1 (i.e. a decrease in surface roughness), should decrease the power of the Schmidt number and near it to the 0.33 value for smooth surfaces.

Surface R2 presents in Fig. 4 a different but complementary behaviour to that observed for surface R1. Due to the relatively high e value, data for surface R2 only include the transition and fully rough regions in the experimental interval of Reynolds numbers studied. However, mass transfer coefficients approach the smooth values as the Reynolds number decreases, suggesting that the smooth regime would eventually be reached if the Reynolds numbers were further decreased. The not collapsed experimental results of R2 indicate, even more clearly than for surface R1, that mass transfer at rough rotating cylinders depends more on the Schmidt number than for smooth cylinders. The $Sh \propto Sc^{0.44 \pm 0.01}$ found for R2 over

$7.5 \times 10^5 > Re > 10^5$ is in very good agreement with previous duct flow mass transfer results obtained for similar rough surfaces [10].

The Schmidt number dependence of the Sherwood numbers determined in the rough surfaces R1 and R2 has been quantitatively discussed. The variation of the transfer rates with Reynolds number is best illustrated in Fig. 5 where the Stanton number is plotted against the Reynolds number for $Sc = 2100$. The three transfer regions mentioned before as the hydraulically smooth transition and fully rough are clearly distinguished in this figure. While data for surface R1 illustrate the first two regions, those for surface R2 show the behaviour in the last two.

Mass transfer coefficients corresponding to surface R1 are coincident in Fig. 5 with smooth data for Reynolds numbers up to $Re \approx 1.3 \times 10^5$, where the hydraulically smooth region ends. This region is not

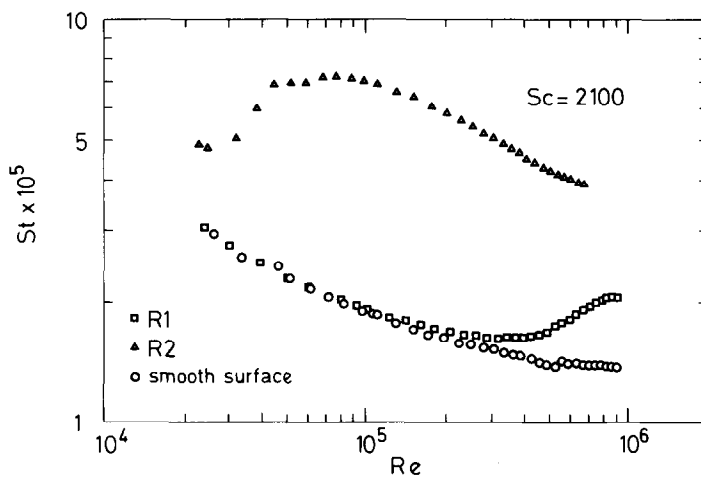


FIG. 5. Variation of the Stanton number with Reynolds number at smooth and rough rotating cylinders.

observed in surface R2 because of its large roughness and high Re studied. A further increase in the Reynolds number forces the Stanton number of R1 to increase until it reaches a maximum value. This transition region is also observed for surface R2 but at lower Reynolds numbers, because the onset of transition occurs at lower Reynolds number for larger roughness elements. At higher Reynolds numbers, in the fully rough region, the Stanton numbers for surface R2 decrease monotonically and with a larger negative slope than for the smooth case. These trends as well as the relation $St \propto Re^{-0.34}$ found for the fully rough region in R2 are in reasonable agreement with results obtained previously in Poiseuille flows [9, 10, 21]. At this point it is important to note that the maximums in the Stanton number distributions shown in Fig. 5 are very flat. For surface R2 the Stanton number does not vary more than 4% about the maximum value in the interval $5 \times 10^4 < Re < 1.1 \times 10^5$. This could explain why in some previous investigations [5–7] concerning circular Couette flows it was claimed, in contradiction with the results obtained in Poiseuille flows, that the Stanton number remained constant in the fully rough region. The present results for R2 and for R1 at $Sc = 770$ indicate that despite the appearance of a flat maximum in the St vs Re distribution, the fully rough region is only reached when the Stanton number decreases faster with Reynolds number than for a smooth surface.

The systematic agreement found between the present results for a circular Couette flow and those published for Poiseuille flows, suggests that there should exist a close relation between the mass transfer results in both types of flows. To this end, mass transfer coefficients for two surfaces practically equal in surface roughness parameters to the R1 and R2, were also determined in a duct flow using the electrolytic method. The characteristics of the equipment used as

well as any experimental detail is given elsewhere [14]. Figure 6 shows the variation of the normalized dimensionless mass transfer coefficients $Sh/Sc^{0.33}$ with Reynolds number determined for R1 and R2 in a duct flow, at $Sc = 550$ and 2100. Independent of the values of the mass transfer coefficients and Reynolds numbers, the trends of the duct flow data for R1 and R2 are very similar to those included in Fig. 4 for the circular Couette flow.

If it is assumed that, at least in the fully rough region where the flow and transfer near the surface are dominated by the presence of the roughness elements, an analogy between Poiseuille and circular Couette flows exists, then the characteristic velocity and length scales representative of the local flow conditions should be the roughness height and the maximum velocity of the flow. Figure 7 shows the variation of the Stanton number with Reynolds number, both normalized with respect to local conditions, determined for surface R1 and R2 at $Sc = 2100$ in the circular Couette and duct flow. The Stanton number variations for R2 match each other, indicating that a quantitative analogy between both flow systems can be drawn for mass transfer in the fully rough region, whenever proper velocity and length scales are used. A similar result, although not so conclusive because fully rough conditions are not well established, is also observed in Fig. 7 for surface R1. As a consequence, previous models and correlations published in the literature for fully rough mass transfer conditions in Poiseuille flows, could be applied to circular Couette flows for similar surfaces.

CONCLUSIONS

An experimental investigation has been carried out to characterize mass transfer at smooth and rough surfaces in a circular Couette flow at high Schmidt and Reynolds numbers. Mass transfer measurements

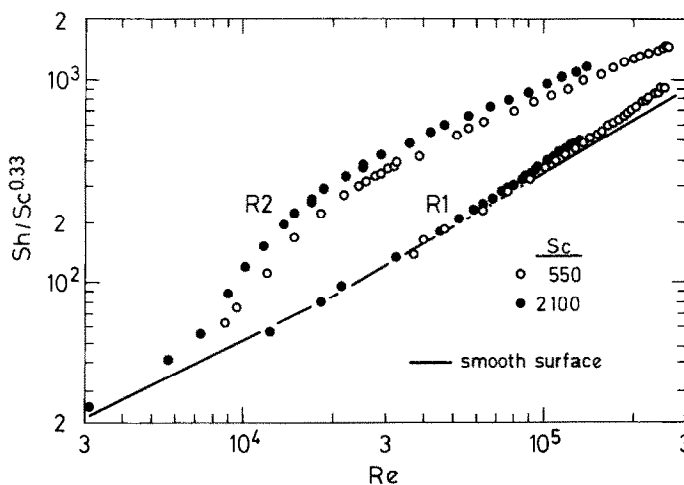


FIG. 6. Variation of the dimensionless mass transfer coefficients with Reynolds number at rough surfaces R1 and R2 in a duct flow.

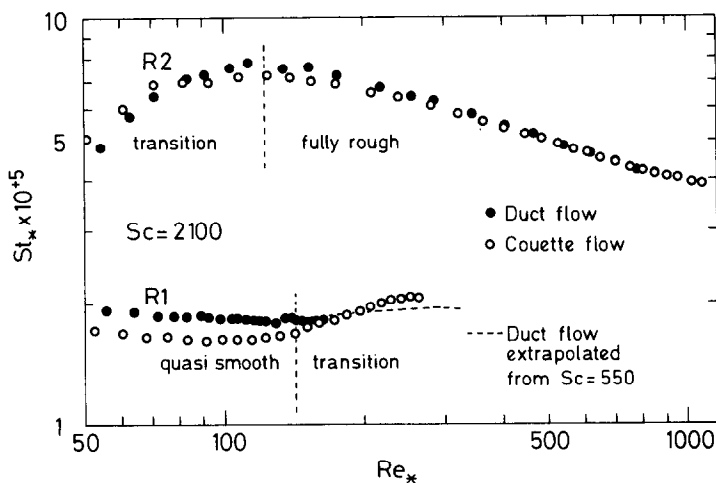


FIG. 7. Generalized mass transfer results for rough surfaces R1 and R2.

show that for smooth rotating cylinders the Sherwood number depends on the 0.33 power of the Schmidt number, as is generally the case for turbulent transport near walls at high Schmidt numbers. Semi-empirical models for transfer in a circular Couette flow cannot be well established because they depend on friction factor values, which are scarce and greatly scattered for this flow system. However, predictions obtained from a semi-empirical model with available friction coefficients, are in reasonable agreement with present and previously published data.

The mass transfer results obtained for the two rough surfaces studied, indicate that in circular Couette flows there also exist the hydraulically smooth, the transition and the fully rough regions found in Poiseuille flows. In the transition region Stanton numbers deviate progressively from smooth values and begin to increase with rotational speed until a broad region of constant and maximum Stanton number is reached, decreasing thereafter with Reynolds number in the fully rough region. The Schmidt number dependence of the Sherwood number for the two rough surfaces studied is larger than for smooth rotating cylinders, in accordance with results reported for duct and pipe flows. A quantitative concordance exists between the fully rough mass transfer results measured in the circular flow system and those also determined in the present study for the same rough surfaces in a duct flow, whenever all dimensionless quantities are referred to the roughness height and maximum velocity of the flow. These results indicate that models valid for fully rough mass transfer conditions in Poiseuille flows may also be applicable to circular Couette flows.

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TRANSFERT MASSIQUE DE SURFACES LISSES ET RUGUEUSES POUR UN ECOULEMENT CIRCULAIRE DE COUETTE

Résumé—Un ensemble étendu de mesures de transfert massique pour un écoulement circulaire de Couette sur des surfaces lisses et rugueuses a été obtenu pour $20\,000 < Re < 900\,000$ et $770 < Sc < 8000$. Les mesures de transfert de masse pour les cylindres tournants lisses sont en accord avec des résultats antérieurs et elles confirment que le nombre de Sherwood dépend de la puissance $1/3$ du nombre de Schmidt. Les coefficients de transfert massique sur des cylindres rugueux montrent un comportement semblable vis-à-vis du nombre de Reynolds que celui observé pour les écoulements en conduite et les résultats expérimentaux pour les deux systèmes peuvent être généralisés dans la région pleinement rugueuse pour des surfaces identiquement rugueuses lorsque la vitesse maximale de l'écoulement et la hauteur de rugosité de la surface sont prises comme échelles caractéristiques de vitesse et de longueur dans les nombres de Stanton et de Reynolds.

STOFFTRANSPORT IN EINER COUETTE-STRÖMUNG AN GLATTEN UND RAUHEN ZYLINDRISCHEN OBERFLÄCHEN

Zusammenfassung—Es wurden umfangreiche Messungen des Stofftransports in einer Couette-Strömung an glatten und rauhen zylindrischen Oberflächen für $20\,000 < Re < 900\,000$ und $770 < Sc < 8000$ durchgeführt. Stofftransportmessungen an glatten rotierenden Zylindern stimmen mit früheren Ergebnissen überein und bestätigen, daß die Sherwoodzahl mit der Potenz 'ein Drittel' von der Schmidtzahl abhängt. Stofftransportkoeffizienten an rauhen Zylindern zeigen eine ähnliche Abhängigkeit von der Reynoldszahl wie jene in Kanalströmungen. Die experimentellen Ergebnisse beider Systeme können im vollkommen rauhen Gebiet für gleich raue Oberflächen verallgemeinert werden, wenn die maximale Geschwindigkeit der Strömung als charakteristische Geschwindigkeit und die Oberflächenrauhtiefe als charakteristische Länge bei der Bildung der Stanton- und Reynoldszahl verwendet werden.

МАССОПЕРЕНОС НА ГЛАДКИХ И ШЕРОХОВАТЫХ ПОВЕРХНОСТЯХ ПРИ КРУГОВОМ ТЕЧЕНИИ КУЭТТА

Аннотация—Выполнена обширная серия измерений массопереноса на гладких и шероховатых поверхностях для кругового течения Куэтта при $20\,000 < Re < 900\,000$ и $770 < Sc < 8000$. Данные по массопереносу для гладких вращающихся цилиндров хорошо согласуются с ранее полученными результатами и подтверждают тот факт, что число Шервуда зависит от $Sc^{1/3}$. Установлено, что зависимость коэффициентов массопереноса на шероховатых цилиндрах от числа Рейнольдса аналогично той, которая наблюдалась при течениях в трубопроводах; экспериментальные результаты для обеих систем могут быть обобщены в полностью шероховатой области для одинаково шероховатых поверхностей, когда за характерную скорость и масштаб длины в числах Стэнтона и Рейнольдса приняты максимальная скорость и высота выступов шероховатости поверхности.